SWAMI VIVEKANAND ACADEMY

Class – XII – Maths –Test Paper – Date: 24/12/2019

General Instructions :

(i) *All the questions are compulsory.*

(ii) *The question paper consists of 36 questions divided into 4 sections A, B, C, and D.*

(iii) *Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.*

(iv) *There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.*

(v) *Use of calculators is not permitted.*

Section 'A'

Q. 1-Q. 10 are multiple choice type questions. Select the correct option.

1. Let $A = \{1, 2, 3\}$ and consider the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$. Then R is

   (a) reflexive but not symmetric **(b)** reflexive but not transitive
   (c) symmetric and transitive **(d)** neither symmetric, nor transitive **1**

2. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

   (a) [1, 2] **(b)** [–1, 1] **(c)** [0, 1] **(d)** None of these **1**

AI 3. $\int \frac{x^9}{(4x^2+1)^6}dx$ is equal to

   (a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+C$ **(b)** $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+C$ **(c)** $\frac{1}{10x}(1+4)^{-5}+C$ **(d)** $\frac{1}{10}\left(\frac{1}{x^2}+4\right)^{-5}+C$ **1**

4. If A is a skew-symmetric matrix, then A^2 is ________.

   (a) symmetric **(b)** skew symmetric **(c)** $A^2 = A$ **(d)** $A^2 = B$ **1**

5. Which of the following is correct?

   (a) Determinant is a square matrix.
   (b) Determinant is a number associated to a matrix.
   (c) Determinant is a number associated to a square matrix.
   (d) None of these. **1**

6. The function $f(x) = e^{|x|}$ is
   (a) continuous everywhere but not differentiable at $x = 0$
   (b) continuous and differentiable everywhere
   (c) not continuous at $x = 0$
   (d) none of these 1

7. If $y = \log \frac{x}{(1+x)}$, then $\frac{dy}{dx}$ is equal to

   (a) $\frac{x}{(1+x)^2}$ (b) $\frac{1}{(1+x)^2}$ (c) $\frac{2x}{(1+x)^2}$ (d) $\frac{1}{x(1+x)}$ 1

8. The line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$, if the value of m is

   (a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$ 1

9. $f(x) = x^x$ has a stationary point at

   (a) $x = e$ (b) $x = \frac{1}{e}$ (c) $x = 1$ (d) $x = \sqrt{e}$ 1

10. The probability distribution of a discrete random variable X is given below :

X	2	3	4	5
$P(X)$	$5/k$	$7/k$	$9/k$	$11/k$

   (a) 8 (b) 16 (c) 32 (d) 48 1

(Q. 11-Q. 15) Fill in the blanks.

11. The vector equation of the line through the points (3, 4, –7) and (1, –1, 6) is 1

OR

The cartesian equation of the plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=2$ is 1

12. In a LPP if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same .... value. 1

13. If A and B are events such that $P(A/B) = P(B/A)$, then 1

14. The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\cos 2x}\, dx$. is equal to 1

AI 15. The solution of the differential equation $\frac{dy}{dx} = 2^{-y}$ is 1

OR

The order and the degree of differential equation :

$\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)$ is 1

(Q. 16-Q. 20) Answer the following questions.

16. Find the rate of change of the area of a circle with respect to its radius r, when $r = 3$ cm 1

17. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$ then find the value of $P(A|B)$. 1

18. Find the anti-derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$. 1

19. If $P(F) = 0.35$ and $P(E \cup F) = 0.85$ and E and F are independent events. Find $P(E)$. 1

20. Find the value of c in the Rolle's Theorem for the function $f(x) = x^3 - 3x$ in the interval $\left[0, \sqrt{3}\right]$. 1

OR

For the function $f(x)=x+\frac{1}{x},\ x \in [1,3]$, the value of c for the mean value theorem. 1

Section 'B'

21. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$. 2

22. Find the angle between the planes

$$7x + 2y + 6z = 15$$

and

$$3x - y + 10z = 17.$$ 2

23. If $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{5}$, then find $P(\overline{A}/\overline{B})$. 2

OR

Find an anti-derivative (or integral) of the function $\cos 3x$ by the method of inspection. 2

24. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation :

$y = x^2 + 2x + C \quad : \quad y' - 2x - 2 = 0$ 2

25. If $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, then find other two roots. 2

26. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black. 2

OR

Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent? 2

Section 'C'

27. Find the value of the expression $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$ 4

28. Show that the lines $\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$ are coplanar. 4

29. If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, then find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$. 4

OR

If $(x^2 + y^2) = xy$, then find $\frac{dy}{dx}$. 4

30. Find the values of x for which $y = [x(x-2)]^2$ is a strictly increasing function. 4

31. Find the particular solution of the differential equation $(1 - y^2)(1 + \log x)dx + 2xy\, dy = 0$ given that $y = 0$ when $x = 1$. 4

OR

Solve the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$. 4

AI 32. Maximise $Z = x + 2y$

Subject to the constraints

$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

Solve the above LPP graphically. 4

Section 'D'

33. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Find each of the following :

(i) $A + B$ (ii) $A - B$ (iii) $3A - C$ (iv) AB (v) BA 6

OR

AI Using properties of determinants, prove that :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab.$$ 6

34. A point on the hypotenuse of a right triangle is at distances 'a' and 'b' from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$. 6

35. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$. 6

OR

Using the method of integration, find the area of the triangle ABC, coordinate of whose vertices are $A(4, 1)$, $B(6, 6)$ and $C(8, 4)$ 6

36. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then prove that :

(i) $\vec{a} = \pm 2(\vec{b} \times \vec{c})$

(ii) $[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = \pm 1$. 6